

**RECOMMENDED VALUES OF STANDARD FREQUENCIES  
FOR APPLICATIONS INCLUDING THE PRACTICAL REALIZATION  
OF THE METRE AND SECONDARY REPRESENTATIONS OF THE  
DEFINITION OF THE SECOND**

**CALCIUM 40 ION ( $f \approx 411$  THz)**

**$^{40}\text{Ca}^+$ ,  $4s^2S_{1/2} - 3d^2D_{5/2}$  unperturbed optical transition**

**1. Recommended value [1] of the frequency in the CIPM List of Frequencies**

$$f(^{40}\text{Ca}^+) = 411\,042\,129\,776\,400.4 \text{ Hz}$$

equivalent to

$$\lambda(^{40}\text{Ca}^+) = 729\,347\,276.793\,942\,7 \text{ fm,}$$

with a relative standard uncertainty of  $1.8 \times 10^{-15}$ .

This radiation is endorsed by the CCTF as a secondary representation of the definition of the second [1].

**2. Method to establish the recommended value**

A global adjustment of all measurements of frequency ratios published in peer-reviewed publications and available to the CCL-CCTF WGFS was carried out following the methods presented in [2-6].

This adjustment determines the frequency of 14 transitions (see Figure 1) which are either already adopted as secondary representations of the second [6] or considered as candidates for SRS. It took into account 105 measurements, including 33 frequency ratios and 72 absolute frequency measurements (i.e. ratios to the  $^{133}\text{Cs}$  frequency). A total of 483 correlations between these input measurements were estimated and considered in the adjustment. More details on the input data and the processing are provided at [https://webtai.bipm.org/ftp/pub/tai/publication/wgfs/Adjustment\\_2021.html](https://webtai.bipm.org/ftp/pub/tai/publication/wgfs/Adjustment_2021.html). The recommended value is the direct result of the adjustment, rounded as deemed adequate with respect to the recommended uncertainty.

While the results are from a global adjustment, it is of interest to note (see Figure 1) that the  $^{40}\text{Ca}^+$  transition is involved in 6 measurements relative to  $^{133}\text{Cs}$ , and in 1 frequency ratio with an optical transition.

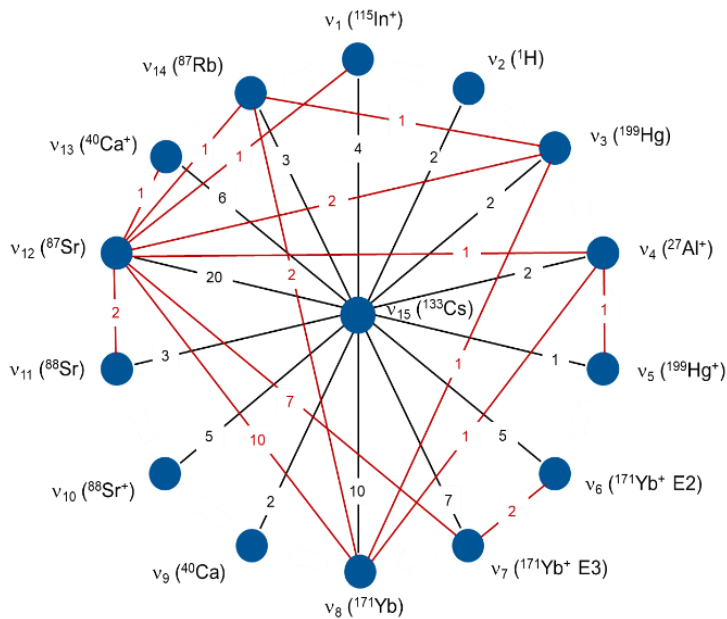


Figure 1: Representation of the 105 measurements linking 14 transitions on the circle and  $^{133}\text{Cs}$  at the center.

### 3. References

- [1] Consultative Committee for Time and Frequency (CCTF), 22<sup>nd</sup> meeting (session II – online), Recommendation PSFS-2 available at <https://www.bipm.org/en/committees/cc/cctf/22-2-2021>
- [2] H. S. Margolis, P. Gill: Least-squares analysis of clock frequency comparison data to deduce optimized frequency and frequency ratio values; *Metrologia* **52**, 628 (2015)
- [3] L. Robertsson: On the evaluation of ultra-high-precision frequency ratio measurements: examining closed loops in a graph theory framework; *Metrologia* **53**, 1272 (2016)
- [4] G. Panfilo, communication to the CCL-CCTF WGFS. A new implementation of [4] was realized in MatLab at the BIPM (2020)
- [5] Ch. Oates: communication to the CCL-CCTF WGFS. An independent program was developed in Mathematica at NIST (2017)
- [6] F. Riehle, P. Gill, F. Arias, L. Robertsson: The CIPM List of Recommended Frequency Standard Values: Guidelines and Procedures; *Metrologia* **55**, 188-200 (2018)