Electrical Units in the New SI: Saying Goodbye to the 1990 Values

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Abstract: The proposed redefinition of several International System (SI) base units is a topic that has been on the metrology agenda for the last decade. Recent progress on several determinations of the fundamental constants means that we now have a good idea of the defined numerical values that will be given in the new system to the Planck constant, \( h \), and the elementary charge, \( e \). This is especially relevant to electrical metrology as new numerical values for the von Klitzing and Josephson constants, given by the relations \( R_K = h/e^2 \) and \( K_J = 2e/h \), will replace the existing 1990 ‘conventional’ values, \( R_{K-90} \) and \( K_{J-90} \). The implementation of the new system cannot be done without introducing small step changes into sizes of the electrical units that are disseminated using Josephson and quantum Hall intrinsic standards. At the time of writing it looks likely that the relative change from \( K_{J-90} \) to \( K_J \) will be of the order \( 1 \times 10^{-7} \), and that from \( R_{K-90} \) to \( R_K \) will be approximately \( 2 \times 10^{-8} \). This paper discusses the practical impact of these changes on electrical metrology and highlights the long term benefits that will come from the updated system.

The CCEM (Consultative Committee for Electricity and Magnetism) of the International Committee for Weights and Measures is now taking the first steps to ensure a smooth implementation, most probably in 2018.

1. Introduction

Discussions on a possible revision of the International System of Units (SI) [1] have been ongoing for over a decade. After an initial concentration on the stability of the kilogram [2, 3], a consensus has emerged for a major revision centered around the redefinition of four of the seven base units, that brings fundamental constants to the fore [4]. Recent progress towards this goal [5] indicates that such a redefinition is now a real possibility for 2018. This paper explores what this upcoming change means for how we realize and disseminate the SI electrical units.

As we will see, no change to working practices or traceability routes is required; the new SI effectively formalizes what is already standard practice in electrical metrology laboratories. In fact, the main change brought about by the new SI is that we will no longer need to worry about the distinction between the ‘representations’ of the volt and the ohm maintained in the laboratory and the presently inaccessible ‘true SI’ units. The new SI will make the present representations of the volt and the ohm equal to the true SI units. This will change very little for most users of electrical calibrations, but the improvement in the overall consistency of the SI is considerable. In the following sections, we review the difficulties with the present situation, before considering the impact of the required changes.

1.1 The Quantum Hall and Josephson Effects

Electrical metrology has had the good fortune to benefit from two remarkable macroscopic quantum effects — the type of physical phenomena that enable us to make the link between the world of fundamental constants and that of everyday calibrations. The quantum Hall effect gives us a quantum standard for electrical resistance \( R_{\text{QHE}} \) [6], via the relation

\[
R_{\text{QHE}} = R_K / n, \tag{1}
\]

and similarly the Josephson effect gives us a quantum standard for voltage \( V_{\text{Jos}} \) [7], via the relation

\[
V_{\text{Jos}} = n f / K_J, \tag{2}
\]

Here \( n \) is an integer and \( f \) the frequency of the radiation driving the Josephson device; \( R_K \) and \( K_J \) are the von Klitzing constant and Josephson constant, respectively.

It is unnecessary to review here more details of these effects or their successful application; however, we note that this success has been a major driver in shaping the proposed revisions of the SI. Two simple relations give the von Klitzing constant and the Josephson constant in terms of the Planck constant, \( h \), and the elementary charge, \( e \),

\[
R_K = h/e^2, \quad \tag{3}
\]

\[
K_J = 2e/h. \tag{4}
\]

Presently we use internationally agreed values (known as the conventional ‘1990 values’, discussed next) in place of the ‘true SI’ values of \( R_K \) and \( K_J \). In the revised SI, we will instead have defined numerical values for the fundamental constants \( h \) and \( e \), and hence, via Eqs. (3) and (4), we will also have defined numerical values for \( R_K \) and \( K_J \). This will make the standards based on the quantum Hall effect and Josephson effect direct realizations of the SI ohm and volt, respectively.
1.2 Electrical Traceability Today: the Conventional 1990 Values

The conventional 1990 values for $R_K$ and $K_J$ were introduced to solve a problem posed by the success of the quantum Hall and the Josephson effects. Resistance and voltage standards based on these effects proved to be extremely precise, repeatable and internationally available, but the sum of experimental results represented in the Committee on Data for Science and Technology (CODATA) Task Group on Fundamental Constants assessments did not allow them to be tied into the SI with anywhere near the same accuracy [8]. The limitations on the accuracy with which the present SI electrical units can be realized is due to the mechanical definition of the base unit, the ampere:

"The ampere is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section, and placed 1 m apart in vacuum, would produce between these conductors a force equal to $2 \times 10^{-7}$ newton per metre of length." [1]

In 1990, the world was not yet ready for a revision of the SI that would abandon this mechanical definition of the ampere, but the new quantum electrical standards were already being widely used due to their near ideal properties. The practical solution chosen was to agree on international fixed values for the constants $R_K$ and $K_J$, known as $R_{K,90}$ and $K_{J,90}$. Whilst this decision ensured that electrical measurements would be consistent world-wide, it meant that units derived from these quantum standards became effectively decoupled from the SI. Details of the considerations leading up to the adoption of $R_{K,90}$ and $K_{J,90}$ and the choice of their values can be found in [8, 9].

The upside to this solution was an enormous gain in the uniformity of primary electrical standards between different national metrology institutes (NMIs). Previous differences in ‘national units’ of up to a few parts in 10^6 were eliminated in a single stroke. The downside of the practical unit realizations being inconsistent with the SI has not turned out to be a major difficulty over the past two decades. It only shows up in comparisons with experiments that have a link to the mechanically defined SI electrical units – essentially watt balances and calculable capacitors. There have probably never been any problems seen in practical calibration work. Still, the discrepancies between the ‘1990 units’ and the SI are clearly not ideal and thus have been the subject of research work, as reviewed in the regular adjustments of the CODATA recommended values for the fundamental constants.

The choice of values for $R_{K,90}$ and $K_{J,90}$ turns out in retrospect to have been sound. The guiding principle at the time was: ‘The values should be so chosen that they are unlikely to require significant change in the foreseeable future’ [8]. After 25 years, we are now ready to put these 1990 values into retirement, and bring the quantum electrical standards fully into the SI. The new SI represents the definitive solution to the problem which the 1990 values temporarily covered over.

The present system of conventional 1990 values also includes additional uncertainties for the rare occasions where use of the true SI is required. In practice, however, these are not often encountered (they are omitted from Calibration and Measurement Capabilities (CMCs), for example). The assigned relative standard uncertainties are $1 \times 10^{-7}$ for the use of $R_{K,90}$ [10] and $4 \times 10^{-7}$ for the use of $K_{J,90}$ [11]. As we shall see in the following sections, the changes we are considering here are substantially smaller than these uncertainties, underlining the validity of the decisions made in 1990.

2. Progress in Knowledge of the Fundamental Constants since 1990

Since 1990, experimental progress has continued on many determinations of fundamental constants and is regularly reviewed in adjustments published by CODATA [12]. In this section, we review what the last 20 years have brought for our knowledge of the constants $R_K$ and $K_J$.

2.1 An Updated Value for $R_K$

The CODATA value for $R_K$ is dominated by experiments that determine the fine structure constant, $\alpha$, due to the relation

$$h/e^2 = \mu_o c/2\alpha, \quad (5)$$

where both the magnetic constant, $\mu_o$, and the speed of light, $c$, have defined numerical values in the present SI. These experiments have improved dramatically in both accuracy and diversity since 1990, leading to the excellent present knowledge of $R_K$. Table 1 shows the successive best estimates over recent years, and these figures are plotted in Fig. 1. We see both an improvement in uncertainty as well as a stable value to within 1 part in 10^8, and thus can predict with reasonable confidence the change on adoption of the new SI. The relative offset from the value of $R_{K,90}$ is around $17 \times 10^{-9}$, and is now believed to be known to better than $1 \times 10^{-9}$.

The graph in Fig. 1 clearly shows one of the challenges of the CODATA adjustments of fundamental constants, namely that new values based on the latest experiments may significantly deviate from previous values (this is likely because uncertainties in experiments have been underestimated). In this specific case, the relative difference between the 2006 and 2010 CODATA values of $R_K$ is $5 \times 10^{-8}$, whereas both values have a relative uncertainty of less than $1 \times 10^{-8}$. All four of the most recent CODATA values of $R_K$ do however agree very well within the level of $1 \times 10^{-8}$, so this discrepancy does not significantly affect the present discussion.
2.2 An Updated Value for $K_J$

The situation for $K_J$ is not as clear as for $R_K$. Table 2 gives the evolution of CODATA values of $K_J$ since 1990 in a similar way to those shown above for $R_K$, and these figures are also plotted in Fig. 2. The data clearly show that there is little improvement in uncertainty, and, more importantly, that the value is not yet reliable at the $10^{-8}$ level.

![Figure 1](image1.png)

**Figure 1.** Data from Table 1: successive CODATA values of the von Klitzing constant $R_K$ published since the adoption of the 1990 value.

The critical experiments in the CODATA adjustments of $K_J$ are watt balance (WB) and silicon sphere (Si28) based Avogadro measurements (see [12] for more details). These are often compared via the effective values of $\hbar$ that the individual experiments give. From Eqs. (3) and (4), we can see that $K_J$ is linked to $\hbar$ and $R_K$ via

$$K_J = \frac{2}{\sqrt{\hbar R_K}}. \quad (6)$$

For the purpose of analyzing the contributions to the value of $K_J$, we can assume the uncertainty on $R_K$ is negligible, and can use Eq. (6) to convert the reported experimental $\hbar$ values to individual values of $K_J$. The results thus obtained for the most important contributing experiments are shown in Fig. 3. The values are plotted as relative differences from $K_{J,90}$ in parts in $10^8$. The uncertainty bars are standard uncertainties.

We plot both results that were included in the latest (2010) CODATA adjustment and those published since [13, 14]. The convergence of recent results predicts a value of $K_J$ approximately 10 parts in $10^8$ below the 1990 value. Although the picture is not yet completely finalized and critical results have only become available within the last six months, it has become clear that we will have to deal with a small, but significant, offset from the 1990 value.

3. Implementation of the New SI – Changes for Electricity

The ampere will remain the base unit for electricity after the proposed revision, and as at present, the dissemination of the electrical units will continue to be based on standards for resistance and voltage using the quantum Hall and Josephson effects. The details of this implementation for the electrical units have been laid out in a draft document, known as the ‘mise en pratique’, that has been available since 2009 [15]. This document (along with equivalents for other areas of metrology) gives the details of how to implement the abstract SI unit definitions in practical realizations. It contains very little of surprise to the electrical metrologist familiar with the present SI. All that has really changed are the two reference values used for $R_K$ and $K_J$.

There will be an inevitable step change in the electrical units realized from quantum standards when this change is implemented, and the numerical values $K_{J,90}$ and $R_{K,90}$ that have been in use for more than 20 years are abrogated and replaced by the new values of $K_J$ and $R_K$ based on the latest experiments.

To understand the impact of this change, we must consider the uncertainties that are achievable in both routine and state of the art measurements today. We consider separately below dc resistance and dc voltage metrology, and finally the wider spectrum of electrical quantities.
3.1 Impact for Resistance Measurements

Whilst quantum Hall resistance (QHR) systems can be compared to the level of 1 part in 10^9 [16], calibrations of travelling resistance standards rarely have relative uncertainties of less than 2 × 10^{-8} due to the limitations of the standards themselves [17]. Consequently, a step change of 0.02 μΩ/Ω in assigned resistor values due to the change from $R_{K-90}$ to $R_K$ should only be seen on the top level working standards maintained within NMIs. Commercial QHR systems have been available for more than 10 years, but have remained relatively complex and expensive, and have not been widely adopted outside of NMIs. This may change in the next few years, as graphene technology promises to significantly simplify QHR equipment [18], but we are not quite there yet. Coordination of the change is thus restricted to NMI experts, and even the most demanding users of resistance traceability will probably be unaffected.

3.2 Impact for Voltage Measurements

Josephson voltage standards have reached a mature level of technological development, and commercial systems are widely distributed into industrial calibration laboratories. We can get a good idea of the state of the art in dc voltage metrology from the North American 10 V Josephson interlaboratory comparison (sponsored by NCSL International). This has been running since 2001, with the latest (9th iteration) completed in 2011. The results and a review of the experiences of 10 years of measurements are reported in [19]. The analysis shows three distinct levels of uncertainty obtainable in voltage comparisons via different techniques. The following figures quoted from [19] are all expanded uncertainties at 95% confidence, given in nV relative to 10 V; the 0.1 μV/V relative change we are considering for $K_J$ is equivalent to 1000 nV in 10 V.

Firstly, direct comparisons of two Josephson systems without any intervening secondary standards can give uncertainties as low as 3 nV. This finding is in line with the experience of the ongoing Bureau International des Poids et Mesures (BIPM) series of on-site Josephson comparisons, conducted world-wide amongst NMIs [20]. Secondly, two Josephson systems in the same lab can be used alternately to measure a common Zener reference. In this case, only the short-term instability of the secondary standard is important, but the uncertainty still rises significantly to the level of 20 nV. Finally, one or more Zener standards can be used as travelling artifacts to compare Josephson systems in separate laboratories. Even with well-characterized Zeners, the transport shocks, the inevitable drift during the time of exchange, and the necessary corrections for conditions of humidity and atmospheric pressure, increase the uncertainty by another factor of 10 to the level of 200 nV.

This final level of uncertainty is also consistent with the CMCs of NMIs offering calibrations of Zener voltage standards directly against Josephson systems, which can indeed be as low as 0.02 μV/V [21]. A step change of five times the uncertainty (at the 95% con-
will thus quickly be lost in the drift line. Normal variations in the medium term, and of 0.1 μV/V in voltage, as outlined in the primary intrinsic standards. The most significant effect will be very marginal beyond the step change in resistance and voltage measured data for very stable standards, this change becomes less worrisome. The results presented in [19] show details of the long-term behaviour of four individual Zener standards over the 10 years during which they have been used for comparisons. They all show drifts in time of order 10 μV/year (not linear over more than a few months, but well fitted by an exponential function). More importantly, the residuals from the fit show rapid and unpredictable variations of amplitude ±1 μV. Figure 4 shows a selected stable Zener that has not travelled, but has been maintained in stable conditions in the BIPM laboratories for the last 10 years. Even here, we see that the step change introduced by the update to the reference value for $K_J$ is of a similar size to normal variations in the medium term, and will thus quickly be lost in the drift line.

### 3.3 Effect beyond Primary Intrinsic Standards

The impact of the new SI and the resulting step change in resistance and voltage measurements will be very marginal beyond the primary intrinsic standards. The most significant effect is a step change of the order of 0.1 μV/V in voltage, as outlined in the previous section. High-end digital voltmeters have specifications of a few parts in $10^{-6}$. Even though they behave better in the well-defined environment of a qualified (national) metrology laboratory, the effect of the step change will still remain unnoticed, swamped by the noise and instability of the internal Zener-diode voltage reference used in the instrument. Other areas of electrical metrology will be essentially unaffected by the envisaged change in voltage, given their uncertainty levels. In the demanding area of primary power measurements, the achieved expanded relative uncertainties of around $2 \times 10^{-6}$ [22, 23] are still an order of magnitude larger than the change in reference value for $K_J$. Capacitance standards are often directly traceable to resistance standards, and in these cases will also see a step change of 0.02 μF/F on the introduction of the updated value for $K_J$. However, even the best calibration uncertainties are larger than this, and the effect will not be visible to end users.

In conclusion, for the wider field of electrical metrology there will be no need for the type of large scale program of education and recalibration that was undertaken for the introduction of the 1990 values (see e.g. [24]).

### 3.4 Implementing the Change

There are a few practical aspects when it comes to implementing the change brought about by the introduction of the new SI. This includes updating the values of $R_K$ and $K_J$ in measurement and data analysis software as well as updating analyses of top level resistance and voltage standards based on their history charts.

For the users of Josephson and QHR systems, implementing the new SI in principle will be as simple as changing one reference number used in the calculation of the measurement results. In practice this can still present some difficulties. The equipment concerned may be a commercial Josephson or QHR system running software supplied by the manufacturer, for which the end-user does not have the source code. A certain amount of time and effort will be required for making these software updates, and that needs planning and coordination. To avoid discrepancies, it is important that the updated software is available at the time of the introduction of the new SI.

As explained above, the step changes introduced into the history graphs of resistors or Zeners will quickly fade into insignificance, but close to the time of redefinition, care must be taken not to mix ‘old’ and ‘new’ values. Drift rates of standards are not affected by the new SI, but in practice, when determining drift rates from measurement data for very stable standards, it is again important not to mix ‘old’ and ‘new’ values; the old values must be corrected for the step change caused by the implementation of the new SI.

The industrial impact of redefinition has been considered previously [25], but we note that the expected change for voltage is now five times larger than considered at that time. Up until now, the details and timing for redefinition have not been sufficiently well developed to start communicating effectively to end users. We can expect this to change over the next few years, with the implementation of the new SI most probably occurring in 2018. The National Institute of Standards and Technology (NIST) in the United States and other NMIs around the world will take up the work of ensuring a smooth transition within industry, and will assess the impact on national calibration infrastructures.

### 4. Conclusions

The revised SI represents a significant step forward for electrical metrology. The electrical units in the new system will be directly linked to the fundamental constants of nature $h$ and $e$ via proven practical quantum standards based on the quantum Hall effect and the Josephson effect. However, in order to get to this position, the widely used conventional 1990 values $K_{90}$ and $R_{K,90}$ must be abandoned and the new values of $R_K$ and
$K$, have to be used. The unavoidable step change introduced on doing so will be at the limits of visibility for the most demanding users of resistance and voltage calibrations. The most significant impact will be in the area of voltage, where a step change of around 0.1 μV/V is foreseen, which will be clearly visible in comparisons of Josephson systems, such as those organized by NCSL International.

This paper is part of the work of a task group of the Consultative Committee on Electricity and Magnetism (CCEM), created to address the implementation of the new SI. The details given here will continue to be updated with new experimental evidence, and the exact changes to be applied will not be known until just prior to the date of implementation (most likely 2018). However, the recent progress means we now have a good picture, and that we can start to prepare for the necessary changes.

5. References


