

Supplements to the Guide to the Expression of Uncertainty in Measurement

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Guide to the Expression of Uncertainty of Measurement

Maintenance by JCGM

Preparation of

Supplements to the GUM

Supporting documents

GUM

Model of measurement

Input quantity values expressed in terms of

Expectations (**best estimates**)

Standard deviations (**standard uncertainties**)

Propagates these through (linearised) model:

law of propagation of uncertainty

Assigns Gaussian (or scaled and shifted t) distribution to output quantity value: **Central Limit Theorem**

Uses this distribution to form **coverage interval**

Approach widely (but not always) applicable

JCGM

Comprises 8 organisations (BIPM, OIML, ISO, IEC, IUPAC, IUPAP, IFCC, ILAC) + NMI experts

JCGM/WG1, Measurement Uncertainty

Responsible for **maintaining and promoting GUM**

Make **no change** to GUM in foreseeable future

Give **added value**, and **address limitations**, through supplements

JCGM/WG2, International Vocabulary of Basic and General Terms in Metrology

Revising and promoting the VIM

Supplements to the GUM

- 1. Propagation of distributions using a Monte Carlo method**
Draft currently being revised following extensive comments
- 2. Models having more than one output quantity**
- 3. Conformance testing**
- 4. Modelling**
- 5. Least squares adjustment**
 - A. Introductory guide**
 - B. Concepts and basic principles**

Two phases of uncertainty evaluation

1. **Formulation** (metrologist)

Model a measurement functionally and probabilistically

2. **Calculation** (statistician, numerical analyst, software)

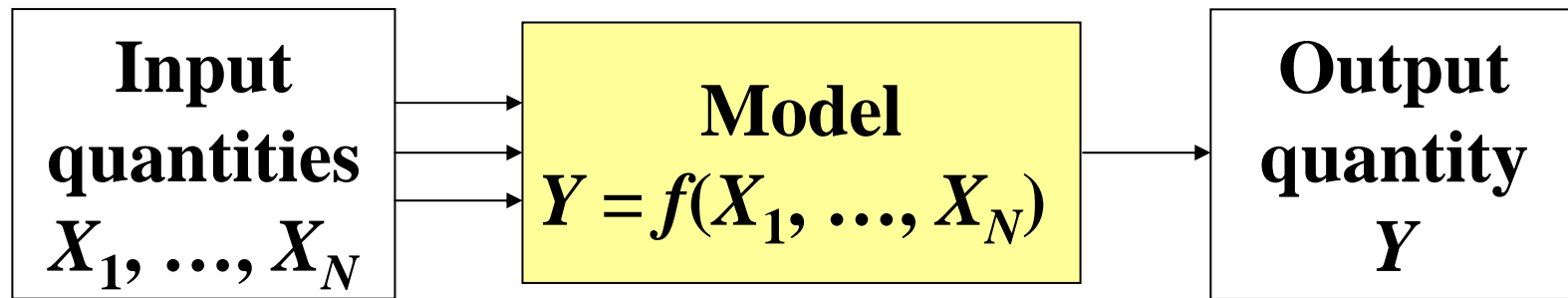
Provide required information:

Estimate of output quantity value

Associated standard uncertainty

Coverage interval

Measurement model

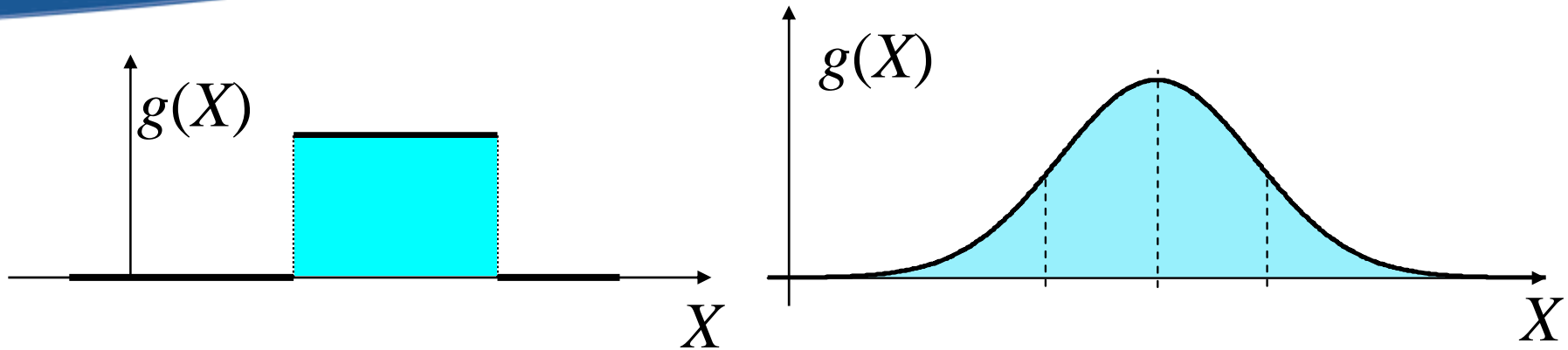


For estimates x_i of values of X_i , model delivers estimate y of value of Y

Because values of X_i known inexactly, value of Y also known inexactly

Uncertainty evaluation is concerned with **quantifying the inexactness of the value of Y**

Probability density functions (PDFs)



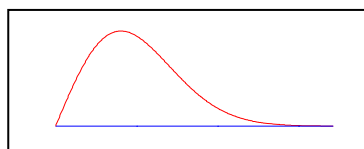
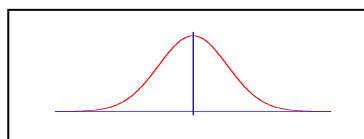
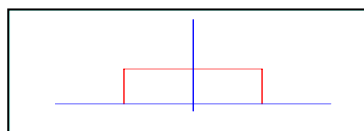
Used to describe **knowledge** about random variable X

For output quantity Y , used to evaluate

- estimate y of value of Y (expectation)
- standard uncertainty $u(y)$ associated with y (standard deviation)
- (shortest or other) coverage interval for value of Y

Propagation of distributions

PDFs for values of X_i



X_1

X_2

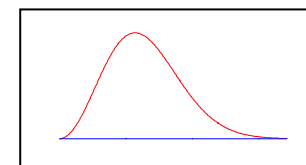
X_3

Measurement model

$$Y = f(X)$$

(X_1, X_2, \dots)

PDF for value of Y



**Formulation phase
(metrological decisions)**

**Calculation phase
(mathematical problem)**

Supplement 1— Propagation of distributions using a Monte Carlo method

Propagates PDFs for values of input quantities X_i through model

Yields PDF for value of output quantity Y

Obtains **all information** required from that PDF

Gives advice on assigning PDFs for values of X_i

Provides **Monte Carlo** implementation

Includes adaptive procedure

Supplement 2 — Models having more than one output quantity

Output quantity Y **vector-valued**

Elements Y_i of Y **almost invariably correlated**

Formulae in GUM **generalized** to this vector case

Expressed in **matrix-vector notation**

For ease of interpretation

As basis for their implementation in software

Supplement 3 — Conformance testing

Relevant to manufacturing industry, health, safety, etc.

Need decisions concerning **characteristics** required to **satisfy specified limits**, but obtained in manner that means **knowledge of them is probabilistic**

Addresses problem of calculating the probabilities that a

(a) characteristic conforms

(b) nonconforming characteristic accepted as conforming

(c) conforming characteristic rejected as nonconforming

Treats formulation stage of uncertainty evaluation

Accepts there are many application-specific aspects

Gives **general guidance on several aspects including**

- (a) **nature of model** (physical, empirical, hybrid)**
- (b) **uncertainty calculation** according to whether
 - (i) quantities are **real or complex****
 - (ii) model is **explicit or implicit****
 - (iii) output quantity is **scalar or vector******

Supplement 5 — Least squares adjustment

Applies to **generalization** of problem addressed by Supplement 2

More model equations than output quantities

LSA used to provide **reduced model** with as many equations as output quantities

Such problems widespread in metrology, especially in **calibration**

Document A — Introductory guide

Introduction to

Measurement uncertainty

GUM

GUM Supplements

Indicates **relevance of GUM**

Shows how supplements **support it**

Document B — Concepts and basic principles

Fundamental concepts and principles

Underpins approaches described in

GUM

GUM Supplements

for evaluating and expressing measurement uncertainty

Form of publication

**GUM Supplements 1–5
and supporting documents A and B**

BIPM website

Introductory document A

Evolving **hyper-linked document**

Points to GUM (will not be available electronically)

Links to GUM Supplements

Links to other relevant uncertainty material

Summary 1

Report on work by JCGM on ‘GUM revision’

Various documents being prepared by JCGM

- 1. Propagation of distributions**
 - 2. Models with general number of output quantities**
 - 3. Conformance testing**
 - 4. Modelling**
 - 5. Least squares adjustment**
- A. Introductory document**
 - B. Scientific underpinning**

Summary 2

**JCGM: wide consultation to ensure documents
have broad application**

Representation and review by eight member bodies

Several NMI experts

Many further contacts through members

Academia, industry, government