Evaluation of Regional Key Comparison Data

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The aim of regional key comparisons (RMO KC) is to extend the metrological equivalence over the measurement standards of national metrology institutes (NMIs), which have not participated in CIPM key comparisons (CIPM KC) .

The degree of equivalence of measurement standards of the NMIs participating in RMO KC, is determined in accordance with item T4 of the Technical Supplement to the MRA, with respect to the reference value of CIPM key comparisons (CIPM KCRV), using the results of measurements, obtained at those NMIs that have participated in both comparisons (linking NMIs).

The procedures of evaluating the RMO KC data considered are correlated with the ***Procedure A*** suggested in ***M.G. Cox, Metrologia, vol. 39, pp. 589-595, 2002***

The procedure A is applied when the following conditions are satisfied:

- each NMI provides a measurement of a traveling standard having good stability and the associated uncertainty

- there is no mutual dependence of the institute’s measurements

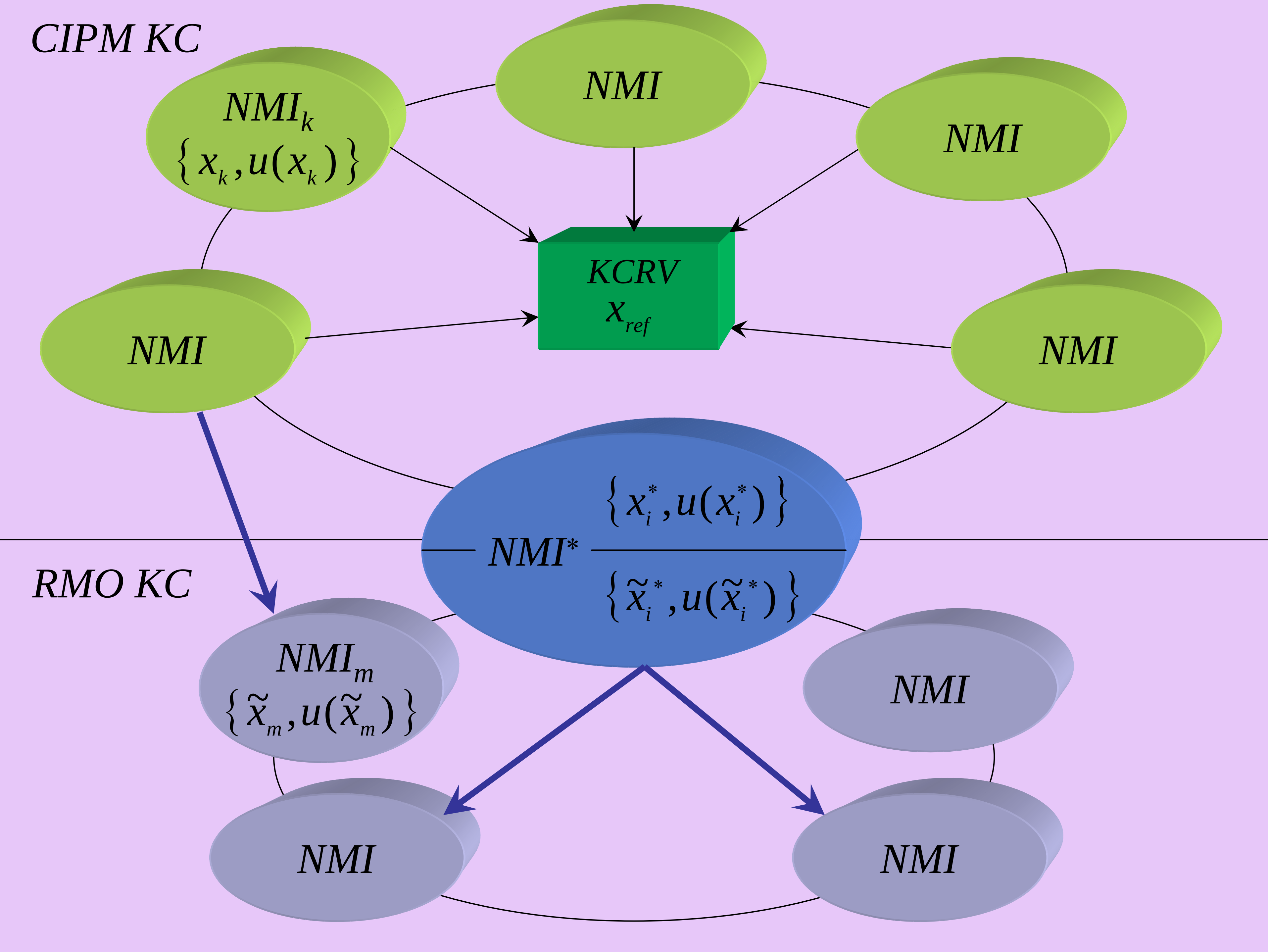
- for each institute a Gaussian distribution can be assigned to the measurand

The first and third conditions are assumed to be valid while the second one seems to be doubles in case of RMO KC.

The following correlations should be taken into account when evaluating RMO KC data:

- the correlations of data obtained in linking laboratories in CIPM and RMO KC

- the correlation of data obtained in laboratories participating in CIPM KC (having primary standards) and data obtained in laboratories participating in RMO KC and borrow the unit from the first ones



CIPM KC
NMI_k
$\{x_k, u(x_k)\}$
NMI
NMI
NMI
NMI
KCRV
x_{ref}
NMI^*
$\{x_i^*, u(x_i^*)\}$
$\{\tilde{x}_i^*, u(\tilde{x}_i^*)\}$
RMO KC
NMI_m
$\{\tilde{x}_m, u(\tilde{x}_m)\}$
NMI
NMI
NMI

In the present paper, two procedures of data evaluation are considered which are denoted below as the “C” and “D” ones conventionally corresponding to two types of comparisons.

The “C” procedure can be applied when the value of a physical quantity is determined. The measure is used as the traveling standard

The “D” procedure can be applied when a value of the calibration coefficient of one and the same measuring instrument used as the traveling standard is determined

It will be shown below that:

- for type C comparisons an additive correction is used and uncertainty calculation in absolute form is applied
- for type D comparison a multiple correction is used and uncertainty reporting in the relative form is more convenient.

Procedure C

The degree of equivalence of the NMI standards that participated in the RMO KC with respect to the KCRV of the CIPM KC is evaluated by formula:

$$d_i = \tilde{x}_i + \Delta - x_{ref}$$

$\tilde{x}_i, \quad i = 1, \ldots, N_1$ are the measurement results obtained by the RMO KC participants, whose number is N1;

Δ - is the additive correction

Xref- is the reference value of CIPM KC (KCRV)

In the case of one linking NMI, the additive correction, Δ, and its standard uncertainty are calculated by formulae :

$$\Delta = x^* - \tilde{x}^*, \qquad u^2(\Delta) = 2S_R^2,$$

Note, that in the case of one linking NMI, the degree of equivalence of the results obtained by the RMO KC is equal to the sum of the degree of equivalence of the linking NMI and the deviation of the result of a RMO KC participant from the result of the linking NMI:

$$d_i = \tilde{x}_i + \Delta - x_{ref} = \tilde{x}_i + x^* - \tilde{x}^* - x_{ref} =$$

$$= \tilde{x}_i - \tilde{x}^* + x^* - x_{ref} = \tilde{x}_i - \tilde{x}^* + d(x^*).$$

In the case of several linking NMIs, the additive correction is calculated as the weighted mean of the corrections calculated using the results of separate linking NMIs :

$$\Delta = \frac{\sum_{i=1}^{L} \frac{\Delta_i}{S_{Ri}^{2}}}{\sum_{i=1}^{L} S_{Ri}^{-2}}, \qquad u^2(\Delta) = \frac{2}{\sum_{i=1}^{L} S_{Ri}^{-2}}$$

where L is the number of linking NMIs.

In a general case of type C comparisons, the standard uncertainty of deviation from the KCRV is evaluated by formula

$$u^2(d_i) = u^2(\tilde{x}_i) + u^2(\Delta) + u^2(x_{ref}) +$$
$$+ 2\operatorname{cov}(\tilde{x}_i, \Delta) - 2\operatorname{cov}(\tilde{x}_i, x_{ref}) - 2\operatorname{cov}(\Delta, x_{ref}).$$

A quantitative assessment of the covariance of measurement results should be done for each particular comparison.

First situation

Measurement results are independent with the exception of the results of linking NMIs. This means that a RMO KC participant does not borrow the unit size from a CIPM KC participant.

Second situation

The measurement results are dependent; this means that a RMO KC participant borrows the unit size from a CIPM KC participant

It is shown that uncertainty of deviation from the KCRV can be calculated using the formula:

"***in first situation***"

$$u^2(d_i) = u^2(\tilde{x}_i) + u^2(x_{ref}) + u^2(\Delta)\left\{1 - u^2\left(x_{ref}\right) \times \sum_1^L u^{-2}\left(x_i^*\right)\right\}$$

« in second situation»

$$u^2(d_i) = u^2(\tilde{x}_i) - u^2(x_{ref}) + u^2(\Delta)\left\{1 - u^2\left(x_{ref}\right) \times \sum_1^L u^{-2}\left(x_i^*\right)\right\}$$

The degree of pair equivalence of the NMIs, participants of the RMO comparisons, is calculated by the formulae:

$$d_{ij} = \tilde{x}_i - \tilde{x}_j ,$$

$$u^2(d_{ij}) = u^2(\tilde{x}_i) + u^2(\tilde{x}_j) - 2\,\mathrm{cov}(\tilde{x}_i, \tilde{x}_j).$$

The degree of pair equivalence between the NMI measurement standards participating in the RMO key comparisons and the NMI measurement standards participating in the CIPM key comparisons is calculated by the formulae

$$d_{ij} = \tilde{x}_i + \Delta - x_j$$

$$u^2(d_{ij}) = u^2(\tilde{x}_i) + u^2(\Delta) + u^2(x_j) +$$
$$+ 2\,\mathrm{cov}(\tilde{x}_i, \Delta) - 2\,\mathrm{cov}(\tilde{x}_i, x_j) - 2\,\mathrm{cov}(x_j, \Delta) =$$
$$= u^2(\tilde{x}_i) + u^2(\Delta) + u^2(x_j) - 2\,\mathrm{cov}(\tilde{x}_i, x_j).$$

Procedure “D”

- In type “D” comparisons, the calibration coefficient of the measuring instruments used as the travelling measurement standards is determined. It is typical of the “D” type comparisons to express the deviation from the KCRV in a relative form:

$$d_{i,rel} = \frac{x_i}{x_{ref}}$$

In the case of one linking NMI the degree of equivalence in a relative form is calculated by the formula:

$$d_{i,rel}(\tilde{x}_i) = \frac{\tilde{x}_i \times \frac{x^*}{\tilde{x}^*}}{x_{ref}} \qquad u^2_{rel}\left(\frac{x^*}{\tilde{x}^*}\right) = 2S^2_{R,rel}$$

in which the ratio of calibration coefficient measurement results obtained by the linking NMI, serves as the multiple correction.

Using some simple transformations it is possible to get the formula for determining the degree of equivalence which in a relative form is the analog of the corresponding formula used in the “C” type comparisons :

$$d_{i,rel}(\tilde{x}_i) = \frac{\tilde{x}_i \times \frac{x^*}{\tilde{x}^*}}{x_{ref}} =$$

$$= \frac{\tilde{x}_i}{\tilde{x}^*} \times \frac{x^*}{x_{ref}} =$$

$$= \frac{\tilde{x}_i}{\tilde{x}^*} \times d_{rel}(x^*)$$

The standard uncertainty of the ratio to the KCRV in relative form is determined by the formulae

$$u^2rel\left(d_{i,rel}\left(\tilde{x}_i\right)\right)=u^2_{rel}\left(\tilde{x}_i\right)+u^2_{rel}\left(\tilde{x}^*\right)+u^2_{rel}\left(x^*\right)+u^2_{rel}\left(x_{ref}\right)+$$
$$+2\operatorname{cov}_{rel}\left(\tilde{x}_i,x^*\right)-2\operatorname{cov}_{rel}\left(\tilde{x}^*,x^*\right)-2\operatorname{cov}_{rel}\left(\tilde{x}_i,x_{ref}\right)+$$
$$+2\operatorname{cov}_{rel}\left(\tilde{x}^*,x_{ref}\right)-2\operatorname{cov}_{rel}\left(\tilde{x}_i,\tilde{x}^*\right)-2\operatorname{cov}_{rel}\left(x^*,x_{ref}\right).$$

where

$$\operatorname{cov}_{rel}(y,z)=\frac{\operatorname{cov}(y,z)}{y\times z}$$

"in first situation"

$$u^2 rel\left(d_{i,rel}\left(\tilde{x}_i\right)\right)=u_{rel}^2\left(\tilde{x}_i\right)+u_{rel}^2\left(x_{ref}\right)+2S_{R,rel}^2\left(1-\frac{u_{rel}^2\left(x_{ref}\right)}{u_{rel}^2\left(x^*\right)}\right)$$

"in second situation"

$$u^2 rel\left(d_{i,rel}\left(\tilde{x}_i\right)\right)=u_{rel}^2\left(\tilde{x}_i\right)-u_{rel}^2\left(x_{ref}\right)+2S_{R,rel}^2\left(1-\frac{u_{rel}^2\left(x_{ref}\right)}{u_{rel}^2\left(x^*\right)}\right)$$

Following the general logic of considering the “D” procedure, it is proposed to calculate the degree of pair equivalence of the NMIs measurement standards participating in the RMO key comparisons, in the form of a ratio:

$$d_{ij,rel} = \frac{d_{i,rel}(\tilde{x}_i)}{d_{j,rel}(\tilde{x}_j)}$$

Simple calculations show that in the case of one linking NMI, the degree of pair - equivalence between RMO KC participants is clearly expressed:

$$d_{ij,rel} = \frac{d_{i,rel}(\tilde{x}_i)}{d_{j,rel}(\tilde{x}_i)} = \frac{\frac{\tilde{x}_i}{x_{ref}} \times \frac{x^*}{\tilde{x}^*}}{\frac{\tilde{x}_j}{x_{ref}} \times \frac{x^*}{\tilde{x}^*}} = \frac{\tilde{x}_i}{\tilde{x}_j}$$

$$u^2_{rel}(d_{ij,rel}) = u^2_{rel}(\tilde{x}_i) + u^2_{rel}(\tilde{x}_j) - 2\mathrm{cov}_{rel}(\tilde{x}_i, \tilde{x}_j)$$

The degree of pair equivalence between the NMI standards participating in RMO KC and NMI standards, participants of the CIPM comparisons, is calculated by formula

$$d_{ij,rel} = \frac{d_{i,rel}(\tilde{x}_i)}{d_{j,rel}(x_j)} = \frac{\frac{\tilde{x}_i}{x_{ref}} \times \frac{x^*}{\tilde{x}^*}}{\frac{x_j}{x_{ref}}} = \frac{\tilde{x}_i}{x_j} \times \frac{x^*}{\tilde{x}^*}$$

$$u_{rel}^2(d_{ij,rel}) = u_{rel}^2(\tilde{x}_i) + u_{rel}^2(x_j) + 2S_{R,rel}^2 - 2\mathrm{cov}_{rel}(\tilde{x}_i, x_j)$$

In the case of several linking NMIs the multiple correction is determined as weighted mean of multiple correction value obtained in every linking lab:

$$c = \frac{\sum_{k=1}^{L} S_{k\,R,rel}^{-2}}{\sum_{k=1}^{L} S_{k\,R,rel}^{-2} \frac{\tilde{x}^{*(k)}}{x^{*(k)}}}$$

Consequently the degree of equivalence in relative form is expressed by the formulae

$$d_{i,rel}(\tilde{x}_i) = \frac{\tilde{x}_i \times c}{x_{ref}}$$

Conclusion

1. The approach for RMO KC data evaluation is discussed . The approach accounts the possible correlation of measurement data

2. It’s suggested to consider two type of comparisons which differ in applying a measure or measuring instrument as the travelling standards

3. Additive and multiple corrections for RMO KC data are proposed.

4. Reporting the degree of equivalence and associated uncertainties in relative form is discussed for comparisons where a calibration coefficient of MI is determined.